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BIOGRAPHY.

KARL FREDERICH GAUSS.

BY B. F. FINKEL.

This versatile and prolific mathematician, Karl Frederich Gauss, was born at Brunswick, Germany, April 30*, 1777, and died at Göttingen on February 23, 1855. His father was a brick-layer and was desirous of profiting by the wages of his son as a laborer, but young Gauss's talents attracted the attention of Bartels, afterwards professor of mathematics at Dorpat, who brought him to the notice of Charles William, Duke of Brunswick. The duke undertook to educate the boy and sent him to the Caroline college, in 1792. By 1795 it was admitted alike by professors and pupils that he knew all that the professors could teach him- It was while at this school that he investigated the method of least squares, and proved by induction the Law of Quadratic Reciprocity. He gave the first rigorous proof of this law and succeeded in discovering eight different demonstrations of it.† While at Caroline college, Gauss manifested as great an aptitude for language as for mathematics, a very general characteristic of eminent mathematicians.

In 1795 Gauss went to Göttingen, as yet undecided whether to pursue philology or mathematics. While at Göttingen he studied mathematics under

*Cf. *Britannica Encyclopedia* and *Century Dictionary*.

†For Gauss's third proof as modified by Dirichlet, see Mathews's *Theory of Numbers*, pages 38-41.

KARL FREDERICH GAUSS.

Abraham Gotthelf Kästner, who was not a very inspiring teacher and who is now chiefly remembered for his *History of Mathematics*, 1796, and by the fact that he was a teacher of the illustrious Gauss. In 1796 he discovered a method of inscribing in a circle a polygon of seventeen sides, and it was this discovery that encouraged him to pursue mathematics rather than philology—a rather insignificant incident to be fraught with such stupendous consequences—consequences materially affecting our present progress in mental and material development.

A detailed construction of this problem by elementary geometry was first made by Pauker and Erchinger.

Gauss worked quite independently of his teachers at Göttingen, and it was while he was there as a student that he made many of his greatest discoveries in the theory of numbers, his favorite subject of investigation. Among his small circle of intimate friends was Wolfgan Bolyai, the discoverer of non-Euclidean geometry.

In 1798 Gauss returned to Brunswick, where he earned a livelihood by private tuition. Later in the year he repaired to the University of Helmstadt to consult the library, and it was while here that he made the acquaintance of Pfaff, a mathematician of great power. Laplace, when asked who was the greatest mathematician in Germany, replied, Pfaff. When the questioner said he should have thought Gauss was, Laplace replied: "Pfaff is the greatest mathematician in Germany; but Gauss is the greatest in all Europe."*

In 1799 Gauss published his demonstration that every algebraical equation with integral coefficients has a root of the form $a+bi$, a theorem of which he gave three distinct proofs. In 1801, he published *Disquisitiones Arithmeticæ*, a work which revolutionized the whole theory of numbers. "The greater part of this most important work was sent to the French Academy the preceding year, and had been rejected with a sneer which, even if the work had been as worthless as the referees believed, would have been unjustifiable."† Gauss had written far in advance of the judges of his work, and so the recognition of its merits had to wait until the mathematical world came in sight of this splendid creation. Gauss was deeply hurt because of this unfortunate incident, and it was partly due to it that he was so reluctant to publish his subsequent investigations.

The next important discovery of Gauss was in a totally different department of mathematics. The absence of a planet between Mars and Jupiter, where Bode's Law would have led observers to expect one, had long been remarked, but not until 1801 was any of the numerous groups of minor planets which occupy that space observed. On the first of January, 1801, Piazzi of Palermo discovered the first of these planets, which he called Ceres, after the tutelary goddess of Sicily.‡ While the announcement of this discovery created no great surprise, yet it was very interesting, since it occurred simultaneously

*Cajori's *A History of Mathematics.*

†Ball's *A Short History of Mathematics.*

‡Young's *General Astronomy*, edition of 1898, page 368.

with a publication by the philosopher Hegel, in which he severely criticised astronomers for not paying more attention to philosophy, a science, said he, which would have shown them at once that there could not possibly be more than seven planets, and a study of which would have prevented, therefore, an absurd waste of time in looking for what in the nature of things could not be found. This is only one instance of the many refutations of dogmatic statements of philosophers who presage nature's laws without confirming them by actual observations.

However, the new planet was seen under conditions so unfavorable as to render it almost impossible to forecast its orbit. Fortunately the observations of the planet were communicated to Gauss. Gauss made use of the fact that six quantities known as elements completely determine the motion of a planet unaffected by perturbations. Since each observation of a planet gives two of these, e. g., the right ascension and declination, therefore three observations are sufficient to determine the six quantities and therefore to completely determine the planet's motion. Gauss applied this method and that of least squares and his his analysis proved a complete success,* the planet being rediscovered at the end of the year in nearly the position indicated by his calculations. This success proved him to be the greatest of astronomers as well as the greatest of "arithmeticians."

The attention excited by these investigations procured for him in 1807 the offer from the Emporer of Russia, of a chair in the Academy ot St. Petersburg. But Gauss, having a marked objection to a mathematical chair, by the advice of the astronomer Olbers, who desired to secure him as director of a proposed new observatory at Göttingen, declined the offer of the emperor and accepted the position at Göttingen. He preferred this position because it afforded him an opportunity to devote all his time to science. He spent his life in Göttingen in the midst of continuous work and after his appointment never slept away from his observatory except on one occasion when he accepted an invitation from Humbolt† and attended a scientific congress at Berlin, in 1828. The only other time that he was absent from Göttingen was in 1854, when a railroad was opened between Göttingen and Hanover.‡

For some years after 1807 his time was almost wholly occupied by work connected with his observatory. In 1809 he published at Hamburg his *Theoria Motus Corporum Coelestium*, a treatise which contributed largely to the improvement of practical astronomy, and introduced the principle of curvilinier triangulation. In this treatise are found four formulæ in spherical trigonometry, commonly called "Gauss's Analogies," but which were published somewhat earlier by Karl Brandon Mollweide of Liepzig, 1774-1825, and still earlier by Jean Baptiste Joseph Delambre (1749-1822).|| On observations in general (1812-1826) we have his memoir, *Theoria Combinationis Observationum Erroribus Minimis Obnoxia*, with a second part and supplement.

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*Berry's *A Short History of Astronomy.*

.†*Britannica Encyclopedia*, 9th edition, Vol. X, page 104.

‡Cajori's *A History of Mathematics.*

||Cajori's *A History of Mathematics.*

A little later he took up the subject of geodesy and from 1821 to 1848 acted as scientific adviser to the Danish and Hanoverian governments for the survey then in progress. His papers of 1843 and 1866, *Ueber Gegenstände der höhern Geodäsie*, contain his researches on the subject.

Gauss's researches on *Electricity and Magnetism* date from about the year 1830. In 1833 he published his first memoir on the theory of magnetism, the title of which is *Intensitas vis Magneticæ Terrestris ad Mensuram Absolutam Revocata*. A few months afterward he, together with Weber, invented the declination instrument and bifilar magnetometer. The same year they erected at Göttingen a magnetic observatory free from iron (as Humbolt and Arago had previously done on a smaller scale), where they made magnetic observations and showed in particular that it was possible and practical to send telegraphic signals, having sent telegraphic signals to neighboring towns. At this observatory he founded an association called the *Magnetische Varein*, composed at first almost entirely of Germans, whose continuous observations at fixed times extended from Holland to Sicily. The volumes of their publications, *Resultate aus der Beobachtungen des Magnetischen Vereins*, extend from 1833 to 1839. In these volumes for 1838 and 1839 are contained two important memoirs by Gauss, one on the general theory of earth-magnetism, the other on the theory of forces attracting according to the inverse squares of the distance. Like Poisson, he treated the phenomena in electrostatics as due to attractions and repulsions between imponderable particles. In electro-dynamics he arrived, in 1835, at a result equivalent to that given by W. E Weber in 1846, viz: that the attraction between two electrified particles, e and e', whose distance apart is r, depends on their relative motion and position according to the formula

$$ee' r^{-2}\{1+(rd^2r-\tfrac{1}{2}dr^2)^2c^{-2}\}.$$

Gauss, however, held that no hypothesis was satisfactory which rested on a formula and was not a consequence of physical conjecture, and as he could not form a plausible physical conjecture he abandoned the subject. Such conjectures were proposed by Riemann in 1858, and by C. Neumann and E. Betti in 1868, but Helmholtz in 1870, 1873 and 1874 showed that these conjectures were untenable.

In 1833, in a memoir on capillary attraction, he solved a problem in the Calculus of Variation, involving the variation of a certain double integral, the limits of integration also being variable; it is the earliest example of the solution of such a problem.

In 1846 was published his *Dioptrische Untersuchungen*, researches on optics, including systems of lenses.

As has already been observed, Gauss's most celebrated work in pure mathematics is the *Disquisitiones Arithmeticæ*, and a new epoch in the theory of numbers dates from the time of its publication. This treatise, Legendre's *Théorie des nombres* and Derichlet's *Vorlesunger über Zahlentheorie* are the standards on the Number Theory.

In this work Gauss has discussed the solution of binominal equations of the form $x^n=1$, which involves the celebrated theorem that the only regular polygons which can be constructed by elementary geometry are those of which the number of sides is $2^m(2^n+1)$, where m and n are integers and 2^n+1 is a prime. These equations are called *cyclotomic equations*, when n is prime and when they are satisfied by a primitive nth root of unity.

Gauss developed the theory of ternary quadratic forms involving two indeterminates, and also investigated the theory of determinants on whose results Jacobi based his researches on this subject.

The theory of Functions of Double Periodicity had its origin in the discoveries of Abel and Jacobi. Both arrived at the Theta Functions which play so large a part in the Theory of Double Periodic Functions. But Gauss had independently and at a far earlier date discovered these functions and their chief properties, having been led to them by certain integrals which occurred in the *Determinatio Attractionis*, to evaluate which he invented the transformation now associated with the name of Jacobi. In the memoir, *Determinatio Attractionis*, it is shown that the secular variations, which the elements of the orbit of a planet experience from the attraction of another planet which disturbs it, are the same as if the mass of the disturbing planet were distributed over its orbit into an elleptic ring in such a manner that equal masses of the ring would correspond to arcs of the orbit described in equal times.

Gauss's collected works have been published by the Royal Society of Göttingen, in seven 4-to volumes, 1863-1871, under the editorship of E. J. Schering. They are as follows: (1) The *Disquisitiones Arithmeticæ*, (2) *Theory of Numbers*, (3) *Analysis*, (4) *Geometry and Method of Least Squares*, (5) *Mathematical Physics*, (6) *Astronomy*, and (7) *Theoria Motus Corporium Cœlestium*. These include besides his various works and memoirs, notices by him of many of these, and of works of other authors in the *Göttingen gelehrte Anzeigen*, and a considerable amount of previously unpublished matter, *Nachlass*. Of the memoirs in pure mathematics, comprised for the most part in volumes ii, iii and iv (but to these must be added those on *Attraction* in volume v), there is not one which has not signally contributed to the branch of mathematics to which it belongs, or which would not require to be carefully analyzed in a history of the subject.

His collected works show that this wonderful mind had touched hidden laws in Mathematics, Physics and Astronomy, and every one of the subjects which he investigated was greatly extended and enriched thereby. He was also well versed in general literature and the chief languages of modern Europe, and was a member of nearly all the leading scientific societies in Europe.

He was the last of the great mathematicians whose interests were nearly universal. Since his time, the literature of most branches of mathematics has grown so rapidly that mathematicians have been forced to specialize in some particular department or departments.

Gauss was a contemporary of Lagrange and Laplace, and these three, of which he was the youngest, were the great masters of modern Analysis. In

Gauss that abundant fertility of invention which was marvelously displayed by the mathematicians of the preceding period, is combined with an absolute rigorousness in demonstration which is too often wanting in their writings. Lagrange was almost faultless both in form and matter, he was careful to explain his procedure, and, though his arguments are general, they are easy to follow. Laplace, on the other hand, explained nothing, was absolutely indifferent to style, and, if satisfied that his results were correct, was content to leave them either without a proof or even a faulty one. Many long and abstruse arguments were passed by with the remark, "it is obvious." This led Dr. Bowditch, of Harvard university, while translating Laplace's *Mechanique Céleste*, to say that whenever he came to Laplace's "it is obvious," he expected to put in about three weeks of hard work in order to see the obviousness. Gauss, in his writings, was as exact and elegant as Lagrange, but even more difficult to follow than Laplace, for he removed every trace of the analysis by which he reached his results, and even studied to give a proof which, while rigorous, should be as concise and synthetical as possible. He said: "Mathematics is the queen of the sciences, and arithmetic is the queen of mathematics," and his *Disquisitiones* confirms the statement.

Gauss had a strong will, and his character showed a curious mixture of self-conscious dignity and child-like simplicity. He was little communicative, and at times morose.

He possessed a remarkable power of attention and concentration, and in this power lies the secret of his wonderful achievements. As a proof of this power of attention we quote from Carpenter's *Mental Physiology*. Gauss, while engaged in one of his most profound investigations, was interrupted by a servant who told him that his wife (to whom he was known to be deeply attached, and who was suffering from a severe illness) was worse. "He seemed to *hear* what was said, but either did not comprehend it or immediately forgot it, and went on with his work. After some little time, the servant came again to say that his mistress was much worse, and to beg that he would come to her at once; to which he replied: 'I will come presently.' Again he lapsed into his previous train of thought, entirely forgetting the intention he had expressed, most probably without having distinctly realized to himself the import either of the communication or of his answer to it. For not long afterwards when the servant came again and assured him that his mistress was dying and that if he did not come immediately he would probably not find her alive, he lifted up his head and calmly replied, 'Tell her to wait until I come,'—a message he had doubtless often before sent when pressed by his wife's request for his presence while he was similarly engaged."

In bringing this imperfect sketch to a close, we wish to call attention to the fact that it has been conclusively shown that Gauss was not the first to give a satisfactory representation of complex numbers in a plane, this having been first satisfactorily done by Casper Wessel in 1797, though Wallis had made some use of graphic representation of complex numbers as early as 1785. Gauss

needs no undue credit to make him famous—the writing alone of any one of the seven of his collected works being sufficient to rank him among the great mathematicians of his day. However, it was Gauss who in 1831, "by means of his great reputation, made the representation of imaginery quantities in the 'Gaussian plane' the common property of all mathematicians." He brought also into general use the sign i for $\sqrt{-1}$, though it was first suggested by Euler. He called $a+bi$ a *complex number* and called a^2+b^2 the *norm*.

THE POPULARIZATION OF NON-EUCLIDEAN GEOMETRY.

By GEORGE BRUCE HALSTED.

In a charming article in the *Popular Science Monthly* for January, 1901, entitled, "Geometry: Ancient and Modern," Edwin S. Crawley delightfully helps the cultured reader to get his orientation in this subject for a start into the new century.

But strangely enough this admirable paper becomes somewhat obscure when it becomes tri-dimensional. It says: "If we proceed beyond the domain of two-dimensional geometry we merge the ideas of non-Euclidean and hyper-space."

If we do so, we are apt to blunder. Just as the Bolyai plane is utterly independent of the Euclidean plane, so the triply extended space of Bolyai is utterly independent of any Euclidean space or hyper-space.

The idea that tri-dimensional Bolyai space needs four-dimensional Euclidean space is an error into which many philosophers and some mathematicians have been led, perhaps from the unfortunate adoption of the name "radius of space curvature" for the space-constant.

This blunder was refuted even before it was born by Bolyai's geodesic geometry of limit surfaces.

Thereby a Euclidean plane can be represented by a surface in Bolyai space, the theorems of Euclidean geometry find their realization as surface theorems in non-Euclidean space, where the geodesic geometry is that of Euclidean straight lines in a Euclidean plane.

Because this error about "curvature in space" is so widespread and so insidious, I treated it fully in my "Report on Progress in Non-Euclidean Geometry" to the American Association for the Advancement of Science.

The very next sentence in Professor Crawley's article reads as follows: "The ordinary triply-extended space of our experience is purely Euclidean."

Here our author states not only something which is not known, but, strangely enough, something which never can be known, which never can be proven.